

Information on the δ^* function

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA. We're going to define a function $\delta^* : \Sigma^* \rightarrow Q$ called the *extended transition function of D* . Intuitively, the function δ^* takes as input a string and outputs what state that string would end up in if run through the DFA D . The function δ^* is defined, recursively, as follows:

- **Base case:** $\delta^*(\epsilon) = q_0$.
- **Recursive case:** If $w \in \Sigma^*$ and $a \in \Sigma$, then $\delta^*(wa) = \delta(\delta^*(w), a)$.

That's quite a mouthful, but there's a nice explanation for what's going on here.

- The base case says that if you power on a DFA and run ϵ through it, you end up in the start state. That makes sense, given that the machine powers on in state q_0 and then doesn't move from there.
- The recursive case says that if we run the string wa through a DFA, we can do so by running w through the DFA, taking us to some state ($\delta^*(w)$), then following the transition labeled a from that state.

We can define $\mathcal{A}(D) = \{ w \in \Sigma^* \mid \delta^*(w) \in F \}$ because the language of a DFA is the set of all strings it accepts, and a DFA accepts a string precisely if it ends up in one of its accepting states. The set given above is the set of all strings w over the DFA's alphabet (that is, strings in Σ^*) where the state w ends up in ($\delta^*(w)$) is accepting (in F).

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA, and let $x, y \in \Sigma^*$ be two strings where $\delta^*(x) = \delta^*(y)$. Prove that, for any string $z \in \Sigma^*$, we have $\delta^*(xz) = \delta^*(yz)$.

In lecture, we used this theorem about distinguishable strings to prove certain languages aren't regular:

Theorem: Let x and y be strings where $x \not\equiv_L y$. Then x and y cannot end up in the same state after being run through any DFA for the language L .

We can recast this theorem in terms of the δ^* function that we just defined above:

Theorem (Formalized): Let x and y be strings where $x \not\equiv_L y$. Then for any DFA D for L , if δ^* is the extended transition function for D , we have $\delta^*(x) \neq \delta^*(y)$.